

Discrete Mathematics

Question (6): $503x \equiv 16 \pmod{761}$

$\Rightarrow$ First, we have to find the inverse of 503 modulo 761

$$\Rightarrow 761 = 1 \cdot 503 + 258$$

$$\Rightarrow 503 = 1 \cdot 258 + 245$$

$$\Rightarrow 258 = 1 \cdot 245 + 13$$

$$\Rightarrow 245 = 18 \cdot 13 + 11$$

$$\Rightarrow 13 = 1 \cdot 11 + 2$$

$$\Rightarrow 11 = 5 \cdot 2 + 1$$

$$\Rightarrow 2 = 1 \cdot 2$$

$$\Rightarrow 1 = 11 - 5 \cdot 2$$

$$= 11 - 5 \cdot (13 - 1 \cdot 11) = -5 \cdot 13 + 6 \cdot 11$$

$$= -5 \cdot 13 + 6 \cdot (245 - 18 \cdot 13) = 6 \cdot 245 - 113 \cdot 13$$

$$= 6 \cdot 245 - 113 \cdot (258 - 1 \cdot 245) = -113 \cdot 258 + 119 \cdot 245$$

$$= -113 \cdot 258 + 119 \cdot (503 - 1 \cdot 258) = 119 \cdot 503 - 232 \cdot 258$$

$$= 119 \cdot 503 - 232 \cdot (761 - 1 \cdot 503) = -232 \cdot 761 + 351 \cdot 503$$

$$\Rightarrow 1 = -232 \cdot 761 + 351 \cdot 503$$

$\downarrow$
inverse

$\Rightarrow$ Thus, 351 is the inverse of 503 modulo 761.

$\Rightarrow$ Multiplying both sides of the congruence by 351 :

$$351 \cdot 503 x \equiv 351 \cdot 16 \pmod{761}$$

$\underbrace{351 \cdot 503}_{5616}$

$\Rightarrow$ Thus, the solutions to the congruence are the integers x such that :

$$\dots, 4094, 4855, 5616, 6377, 7138, \dots$$

$\nwarrow$ $\nearrow$
 -761 $+761$